# CS 295A/395D: Artificial Intelligence

#### Potpourri of Unit 3

Prof. Emma Tosch

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The University of Vermont

## **First: Design pattern in AI/ML**

Many tasks can be boiled down to alternating between:

- 1. Computing an expected value
- 2. Finding an argument (i.e., making a choice) that maximizes some function

For problems of any complexity, that function will be composed of other functions, including random variables.

## Why expected values?

High level view:

Expected values are summary information and are useful when:

- 1. We don't know the point value (its value has either aleatory or epistemic uncertainty)
- 2. BUT...we know its distribution

Can make decisions on the basis of that summary information!

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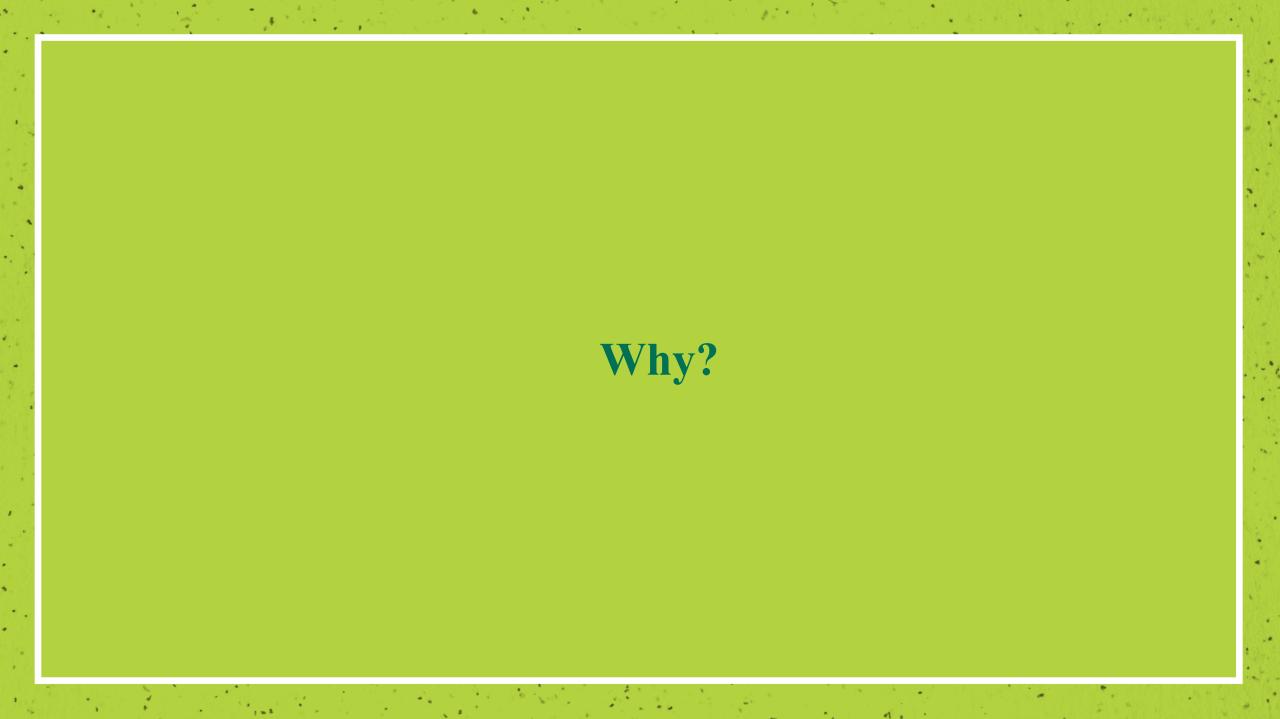
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#### Game theory: recipe for mixed strategies

- 1. Compute expression for  $E[P \mid Q = q]$  using P's utility function.
  - 1. Reminder: this will be over all possible choices for P.
  - 2. The pmf comes from P's choice.
  - 3. Q's choice is fixed here.
- 2. Compute expression for other values of Q=q.
- 3. Set expressions equal to each other and solve for p.



## **Utility functions**

Recall: all agents are "rational"

- All agents have complete knowledge of the payoff matrix
- All agents seek to maximize their utility
- Assumption: all players are treating the game in a decisiontheoretic manner

#### **Can model game theory as decision theory**

Given: simultaneous, no communication or coordination

- Treat other agent's actions random state
- Each state node encapsulates all of the uncertainty about player Q's actions
- Objective: use game theory to determine state distribution

#### **Relation to minimax**

Previously: search in a planning context

- Planning with logic (e.g. STRIPS, PADDL)
  - Search through change state as logical inference over limited language
  - Enforcing constraints
  - Challenge: finding a path efficiently
- Introduced notions of heuristics + cost. Difference?
  - Heuristics are estimates (used when it's okay to be slightly sub-optimal)
  - Costs assign value to state, used for ordering
- All deterministic; here, probabilistic

## **Minimax theorem in game theory**

Subtlety in the player's objective:

- Minimize max loss?
- Maximize min gain?

Assume zero-sum game:

Player X maximizing its minimum gain is equivalent to minimizing its max loss.  $\max \min x^T A u = \min \max x^T A u$ 

$$\max_{x\in X}\min_{y\in Y}x^TAy = \min_{y\in Y}\max_{x\in X}x^TAy$$

## **Recipe for easy solutions**

Recall: Decisions are made locally

- Optimum vs. optimal
  - Optimum is global (something we cannot control)
  - Optimal is local (something we can control)
- Special case: saddle point for zero-sum games
  - Minimum between choices for P (here, between columns)
  - Maximum between choices for Q (here, between rows)

#### **Utility functions: other models?**

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#### Formalizing with epistemic knowledge

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