CS 295A/395D: Artificial Intelligence

Epistemic Logic:

Knowledge & Belief

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“Modes” of truth

Recall: logic with respect to a knowledge base

Contextualizes statements – need abstract notion of context

- Informally, time: when is something true?
  - When must something be true? Refers not to time, but “in what context?”
- Formally more abstract than time: time has a specific sequential meaning, but we may want a broader definition

Recap: Semantics

Recall: a formal *semantics* is a mapping from a surface string (syntax) to an underlying structure that gives a surface string meaning.

Propositional logic example: Given assignment $\mathcal{A} = \{\mathrm{p} : 1, \mathrm{q} : 0, r : 1\}$, $\mathcal{A} \vDash (p \lor q) \longrightarrow r$

Predicate logic example: Given $\mathcal{U} = \mathbb{N}$ and structure $\mathcal{M} = (\mathcal{F} = \{+\},\ \mathcal{P} = \{=\})$, $\mathcal{M} \vDash \forall n \exists m (m = n + 1)$

Finding an assignment that models a formula == searching for a satisfying assignment (SAT)

Finding a structure to model a predicate formula → not emphasized

Basic modal logic: Syntax

Take everything from propositional logic and add:

- □ (“box” = “necessity” → like ∀)
- ◇ (“diamond” = “possibility” → like ∃)

These are unary operators that can be prefixed to any valid propositional or modal logical formula:

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p	$\neg p$	$q \land p$	$p \to q$	$p \lor q$
$\Box p$	$\Diamond \neg p$			

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$\Box\, p$	$\Diamond \neg p$	$\Box(q \wedge p)$	$\Diamond(p \rightarrow q)$	

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$\Box \Box p$	$\neg \Diamond p$			

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$\Box\,\Box\, p$	$\neg \Diamond p$	$\Box q \wedge \Diamond p$	$\Diamond(p \rightarrow \Box q)$	

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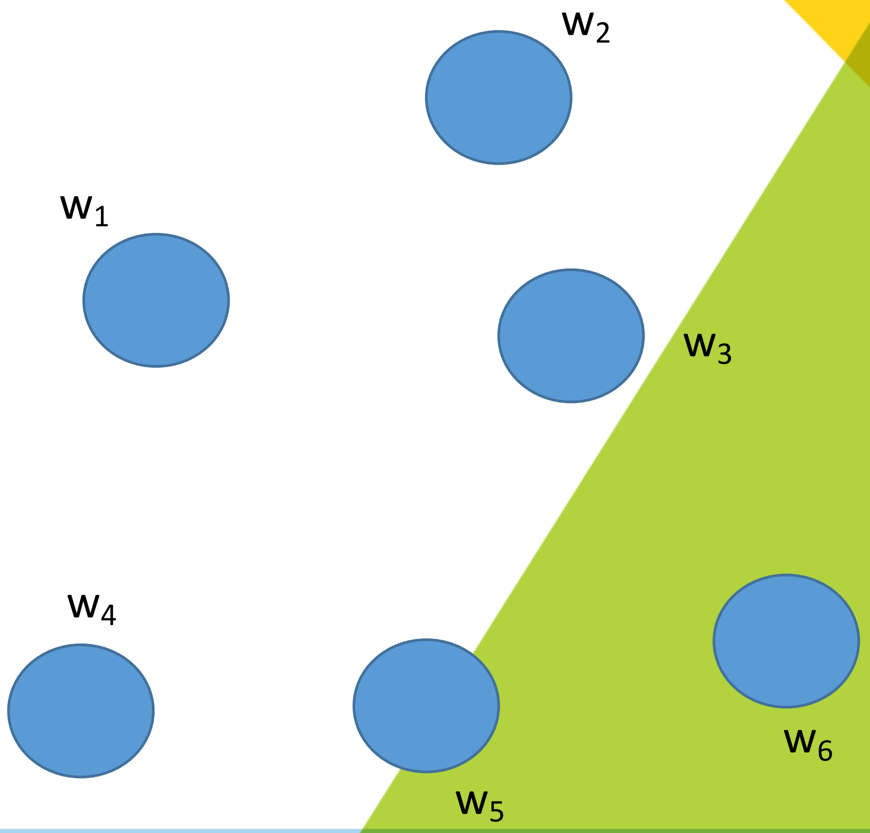
$\Box\, \Box\, p$	$\neg \Diamond p$	$\Box q \wedge \Diamond p$	$\Diamond(p \rightarrow \Box q)$	$\Box \Diamond(q \vee \Box\, p)$

Basic modal logic: Model specification

Model structure: $\mathcal{M} = (W, R, L)$

- W: set of "worlds" (nodes in a graph),



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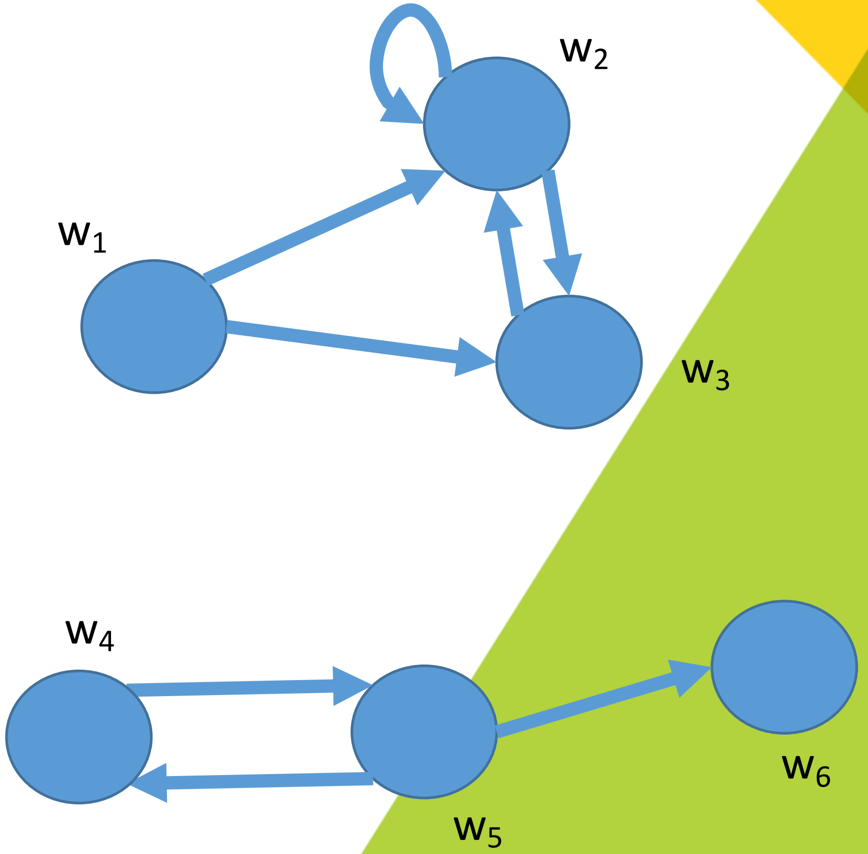
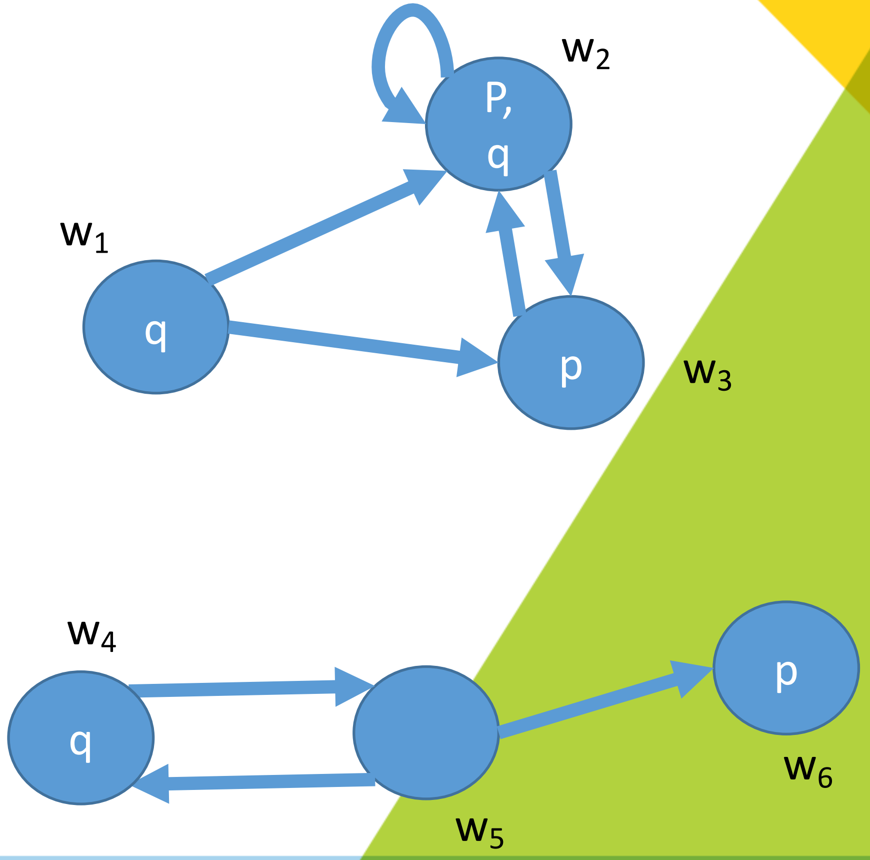


Basic modal logic: Model specification

Model structure: $\mathcal{M} = (W, R, L)$

- W: set of "worlds" (nodes in a graph),
- R: binary "accessibility relation" on W (edges in a graph),
- L: "labeling function" from each world to a subset of atoms,

Where the set of atoms is the set of propositions.



Basic modal logic: Semantics

Given Model structure: $\mathcal{M} = (W, R, L)$. Let $w \in W$. Then:

$w \Vdash \top$

$w \nVdash \bot$

$w \Vdash a$ iff $a \in L(w)$

$w \Vdash \neg\varphi$ iff $w \nVdash \varphi$

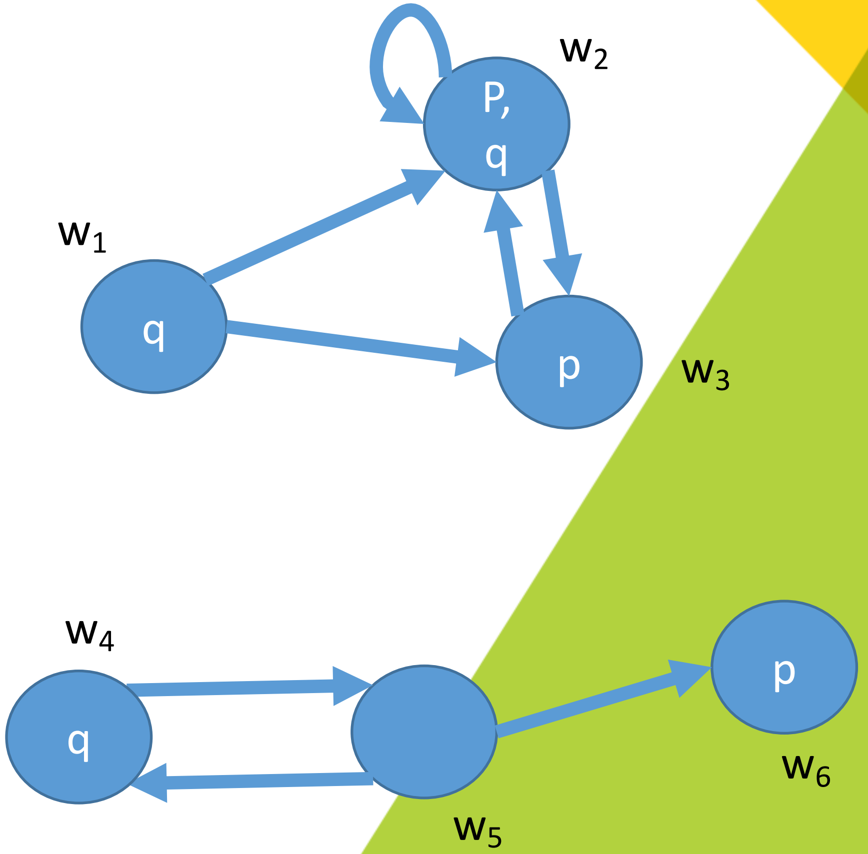
$w \Vdash \varphi \wedge \psi$ iff $w \Vdash \varphi$ and $w \Vdash \psi$

$w \Vdash \varphi \vee \psi$ iff at least one of $w \Vdash \varphi$ or $w \Vdash \psi$

$w \Vdash \varphi \rightarrow \psi$ iff $w \Vdash \varphi$ whenever $w \Vdash \psi$

$w \Vdash \Box\varphi$ iff $\forall\, w' \in W\ (R(w, w') \rightarrow w' \Vdash \psi)$

$w \Vdash \Diamond\varphi$ iff $\exists\, w' \in W\ (R(w, w') \wedge w' \Vdash \psi)$



Let φ = q and ψ = p

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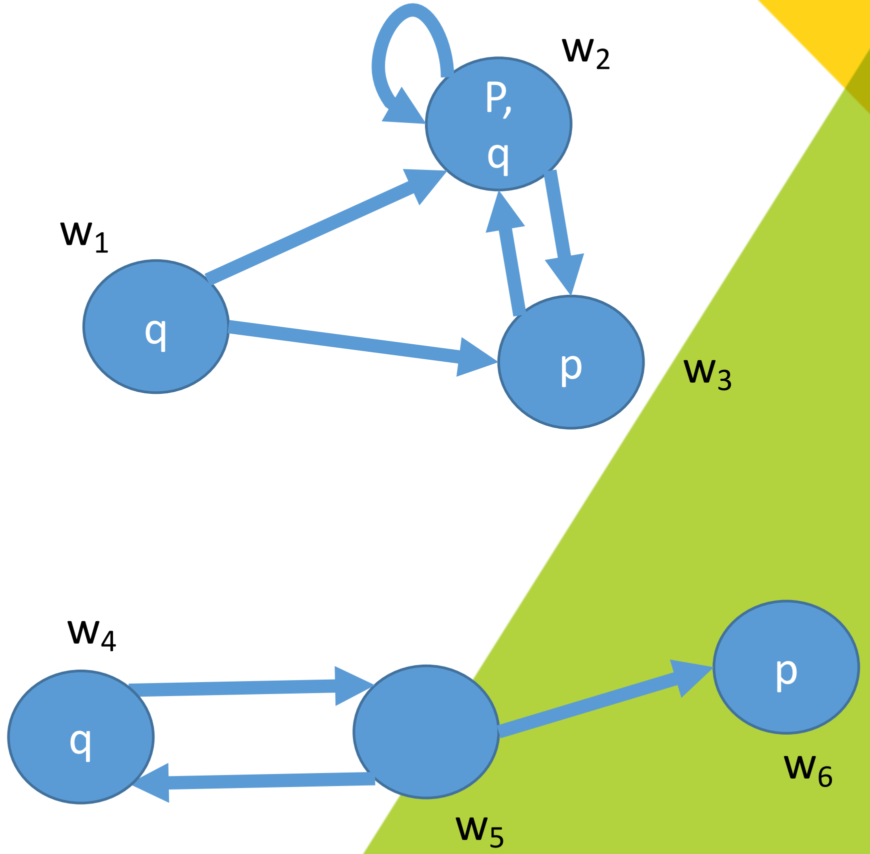

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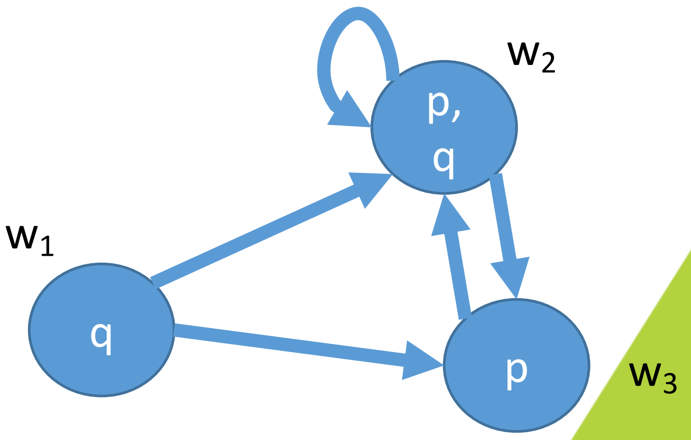

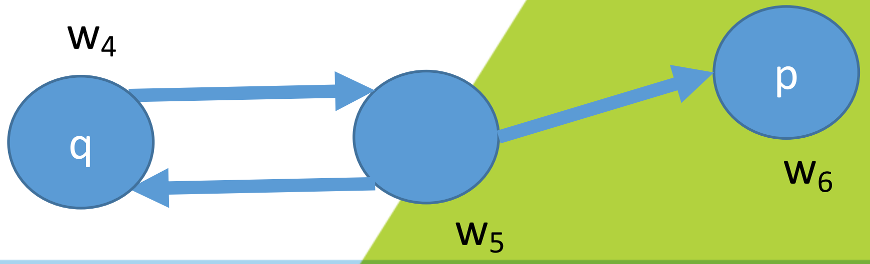

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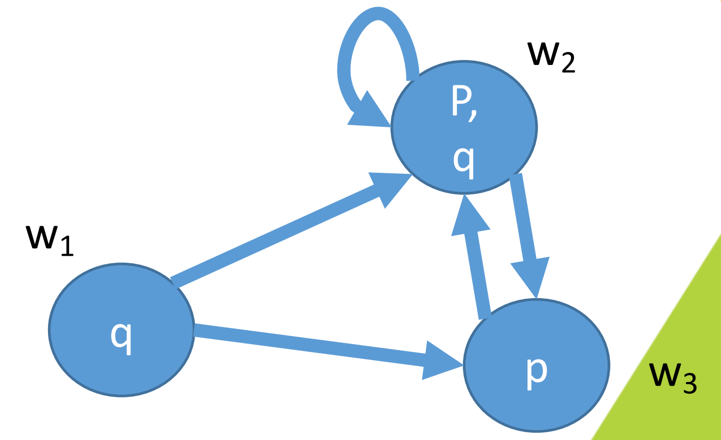

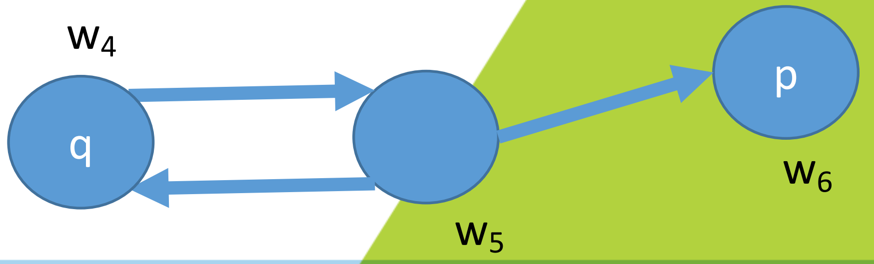

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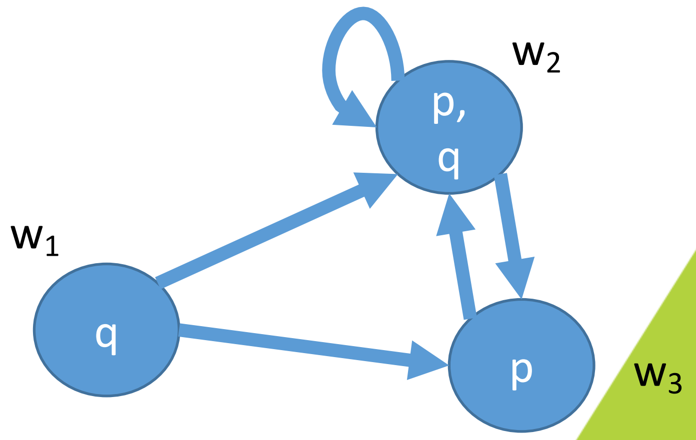

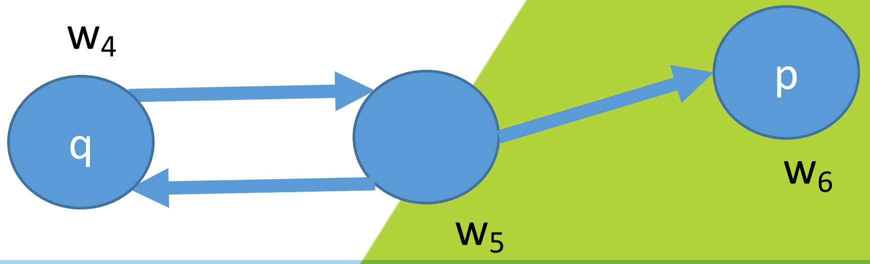

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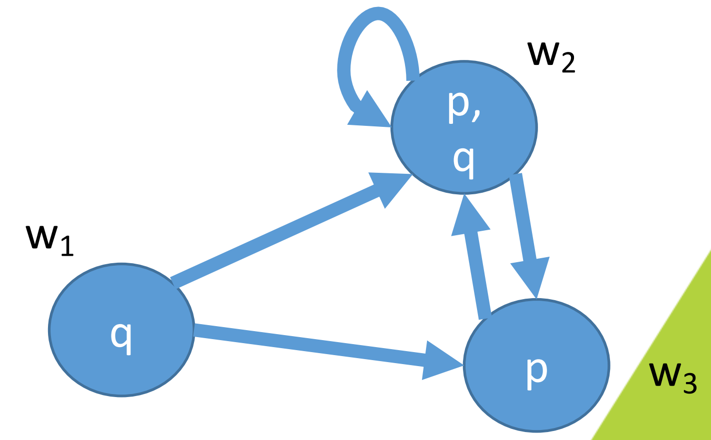

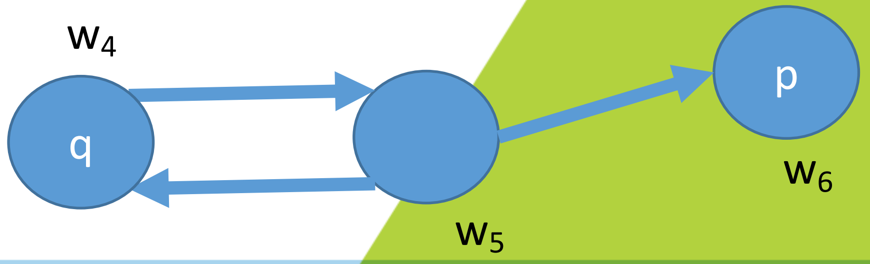

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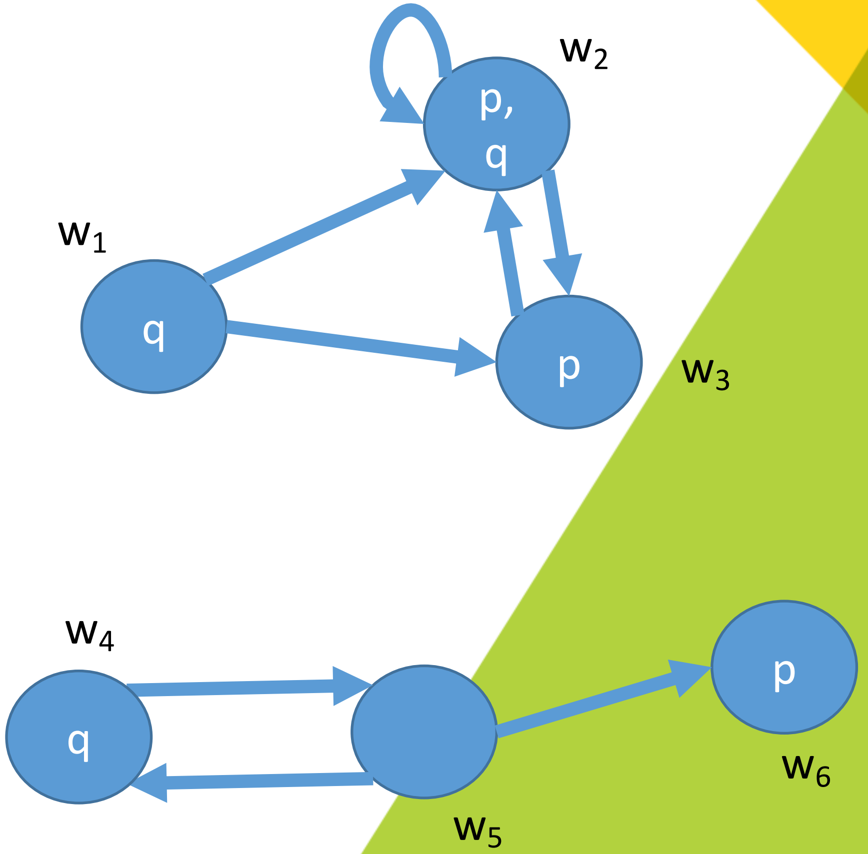

Find a world w such that $w \Vdash \Box p \vee \Box q$

Basic modal logic: Semantics

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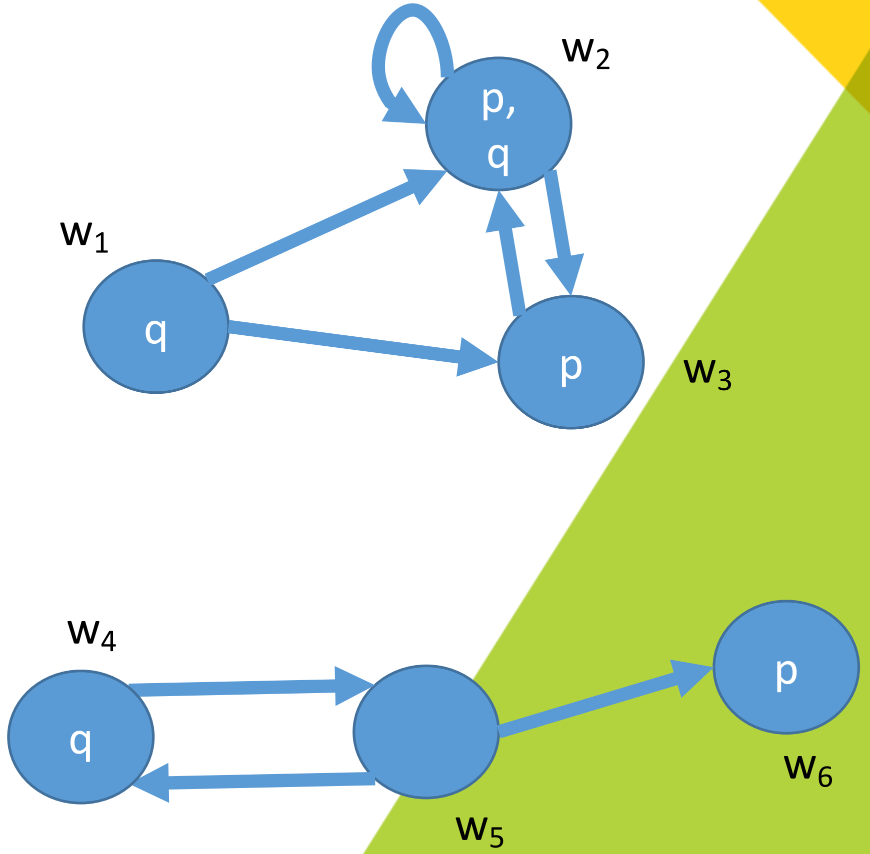

Find a world w such that $w \Vdash \Box(p \vee q)$

Basic modal logic: Semantics

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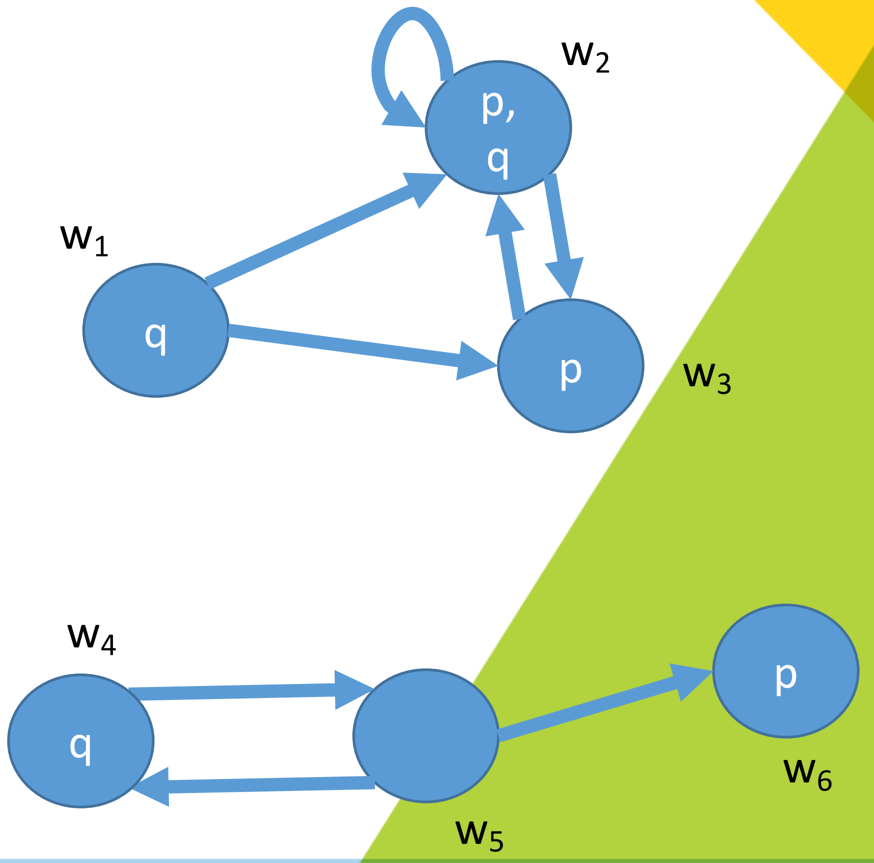

Find a world w such that $w \nVdash \Box p \rightarrow p$

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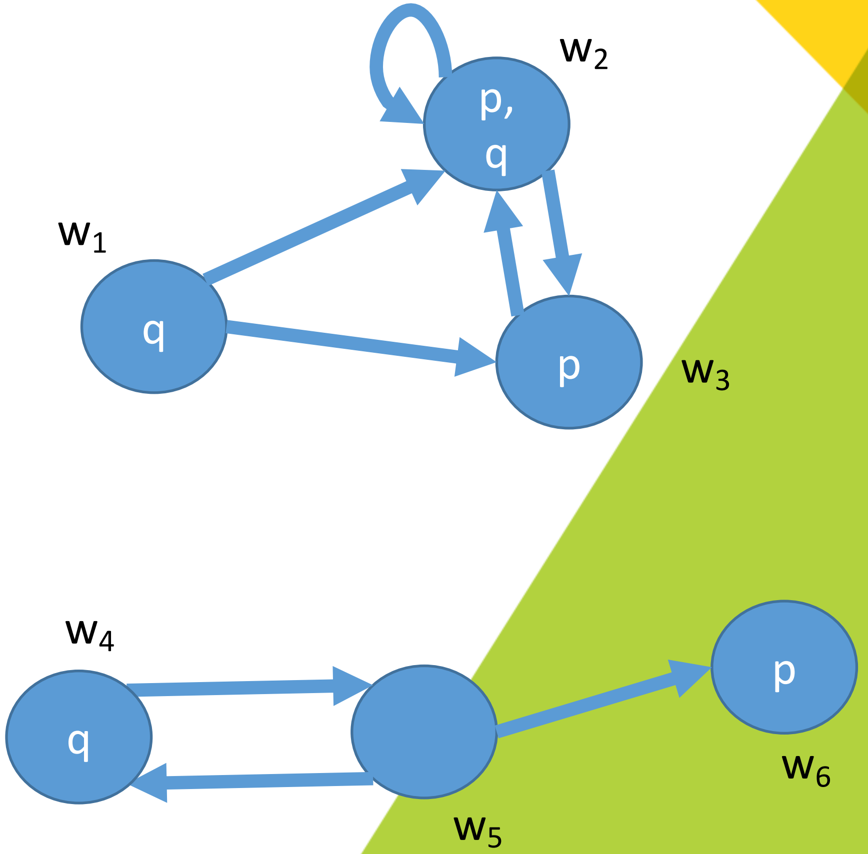

Find a world w such that $w \nVdash \Diamond\top$

Basic modal logic: Semantics

Given Model structure: $\mathcal{M} = (W, R, L)$. Let $w \in W$. Then:

$w \Vdash \top$

$w \nVdash \bot$

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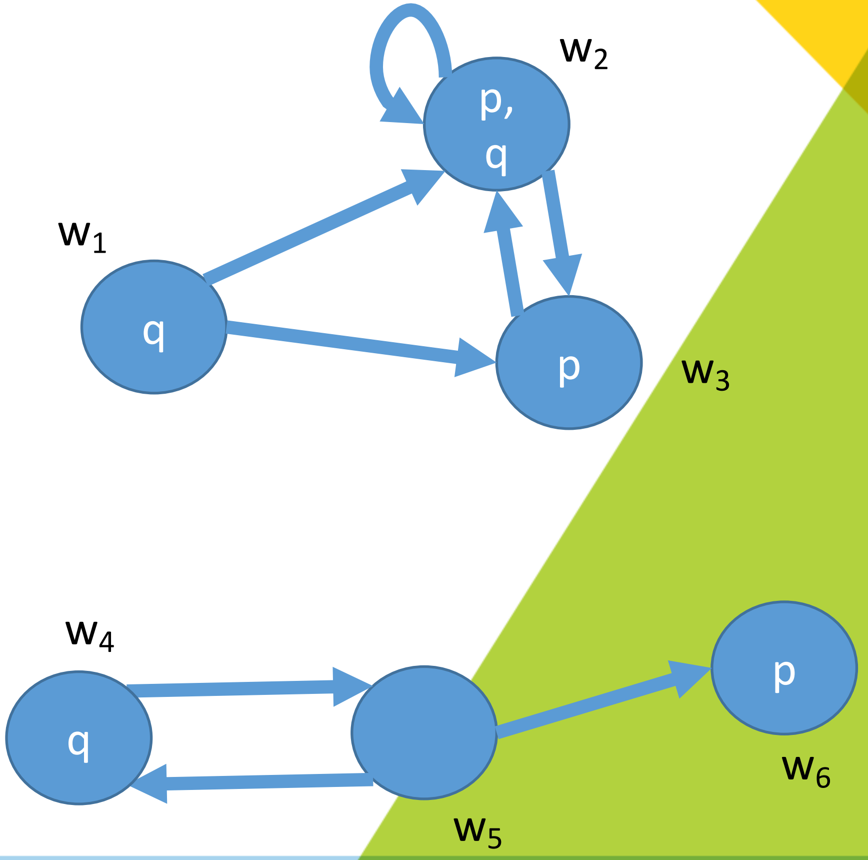

Find a world w such that $w \Vdash \Box\bot$

New rules/equivalences

DeMorgan's:

$$\neg\Box\varphi = \Diamond\neg\varphi$$

$$\neg\Diamond\varphi = \Box\neg\varphi$$

Distributive:

$$\Box(\varphi \land \psi) = \Box\varphi \lor \Box\psi$$

$$\Diamond(\varphi \lor \psi) = \Diamond\varphi \land \Diamond\psi$$

Connective equivalence:

$$\neg\Box\neg\varphi = \Diamond\varphi$$

Tautology, contradiction:

$$\Box\top = \top$$

$$\Box\top \neq \Diamond\top$$

$$\Diamond\bot = \bot$$

$$\Diamond\bot \neq \Box\bot$$

Basic modal logic: Stacked modals

Given Model structure: $\mathcal{M} = (W, R, L)$. Let $w \in W$. Then:

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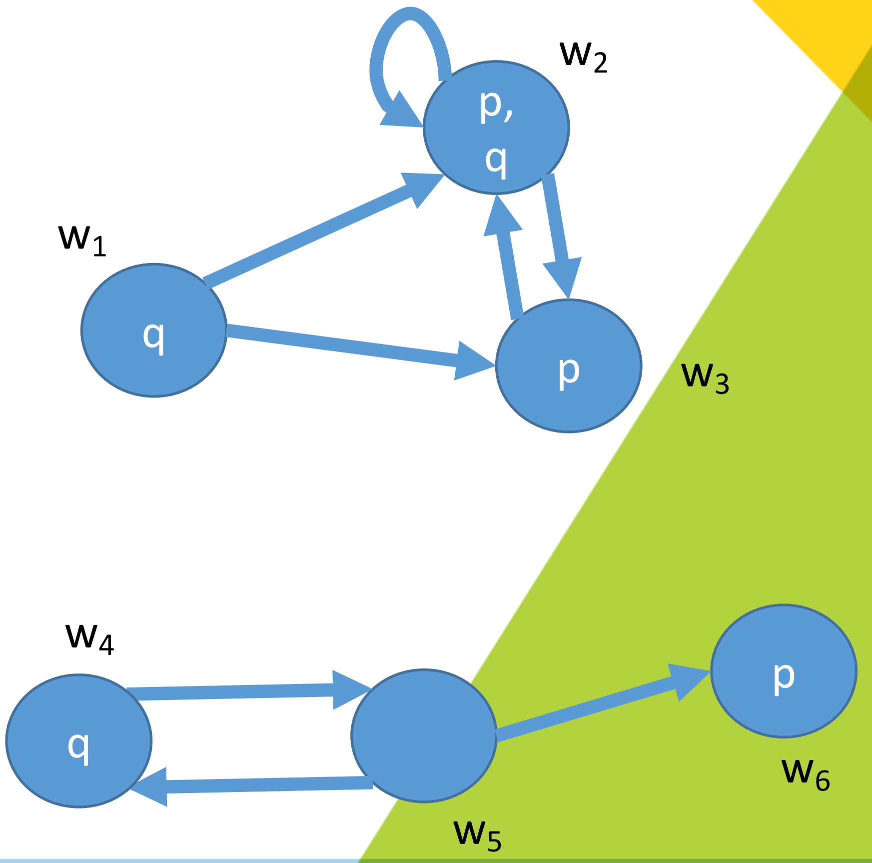

Find a world w such that $w \Vdash \Box \Diamond$ q

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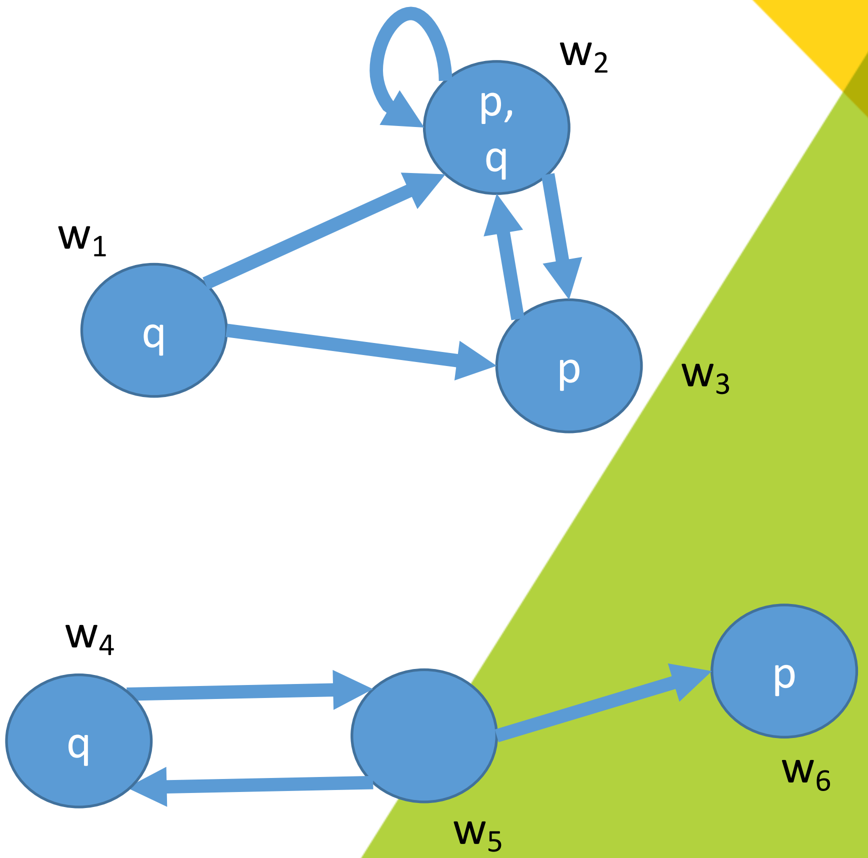

Find a world w such that $w \Vdash \Diamond\Diamond\Diamond \Box \bot$

Model satisfiability

So far: world safisfiability ($\mathrm{e.g.}, w \Vdash p$).

Let $\mathcal{M} = (W, R, L)$. If every world $w \in W$ satisfies a formula φ, then we say $\mathcal{M} \vDash \varphi$.

Valid formulas: Propositional logic

Recall: a formula is valid iff every possible assignment/structure makes it true.

Equivalently: a formula is valid iff there does not exist an assignment that could make it false.

Examples:

$$p \vee \neg p \qquad\qquad p \rightarrow p \qquad\qquad (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$$

Valid formulas: Basic Modal Logic

All the same valid formulas, plus "K":

$$(\Box(\varphi \rightarrow \psi) \land \Box\varphi) \rightarrow \Box\psi$$

Also written:

$$\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$$

Work through derivation on the board

New interpretation of modes: Knowledge, Belief

Necessity, possibility → abstract concept

Can provide alternative interpretations of "It is necessarily true that φ" for the string $\Box\varphi$:

- "It will always be true that φ" (temporal logic → coming up!)
- "It ought to be true that" (deontological → law)
- "Agent Q believes that" (belief)
- "Agent Q knows that" (knowledge)

New interpretation of modes: Knowledge, Belief

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- "Agent Q knows that" (knowledge)

$\diamond\varphi$

"φ is consistent with Q's beliefs"

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$\Box\varphi$

$\Diamond\ \varphi$

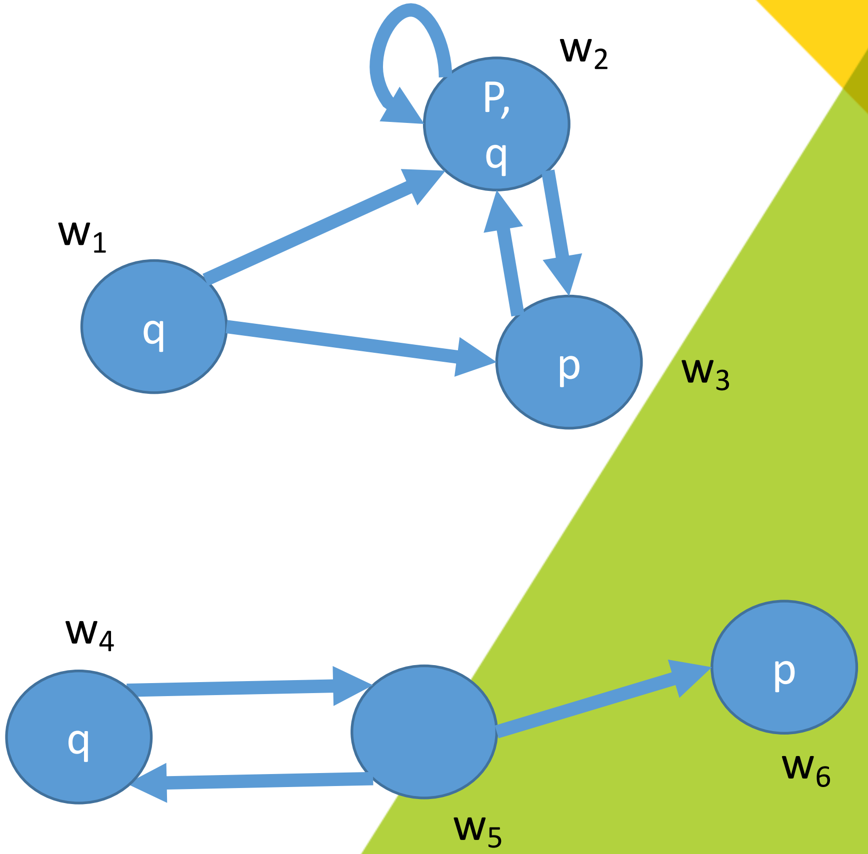
"For all Q knows, φ"

Both interpretations of possible (diamond) are more intuitive under $\neg\Box\neg\varphi = \Diamond\ \varphi$

Capturing domain-specific axioms

Returning to the totally abstracted case:

Is $\Box p \rightarrow p$ valid?



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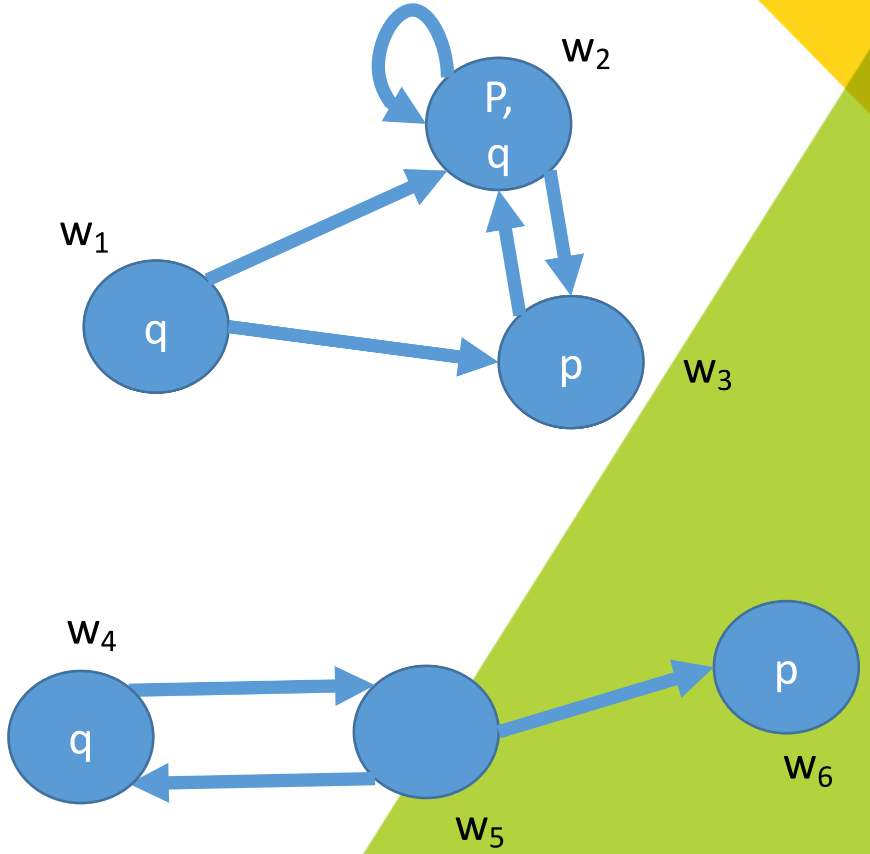
If $\Box$ represents **belief**, is $\Box p \rightarrow p$?

(Recall: $\Box p$ means "Agent Q believes p to be true")



Capturing domain-specific axioms

Returning to the totally abstracted case:

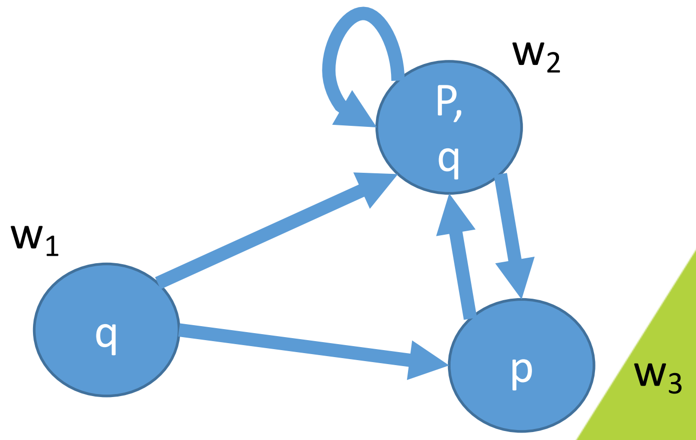
Is $\Box p \rightarrow p$ valid?

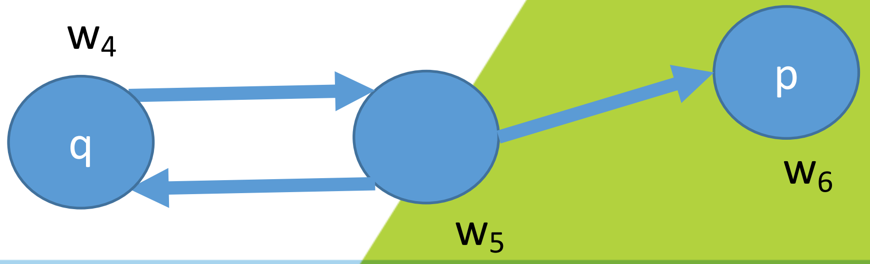
If $\Box$ represents belief, is $\Box p \rightarrow p$?

(Recall: $\Box p$ means "Agent Q believes p to be true")

If $\Box$ represents **knowledge**, is $\Box p \rightarrow p$?

(Recall: $\Box p$ means "Agent Q knows p to be true")





Sometimes we want to assert a formula schema

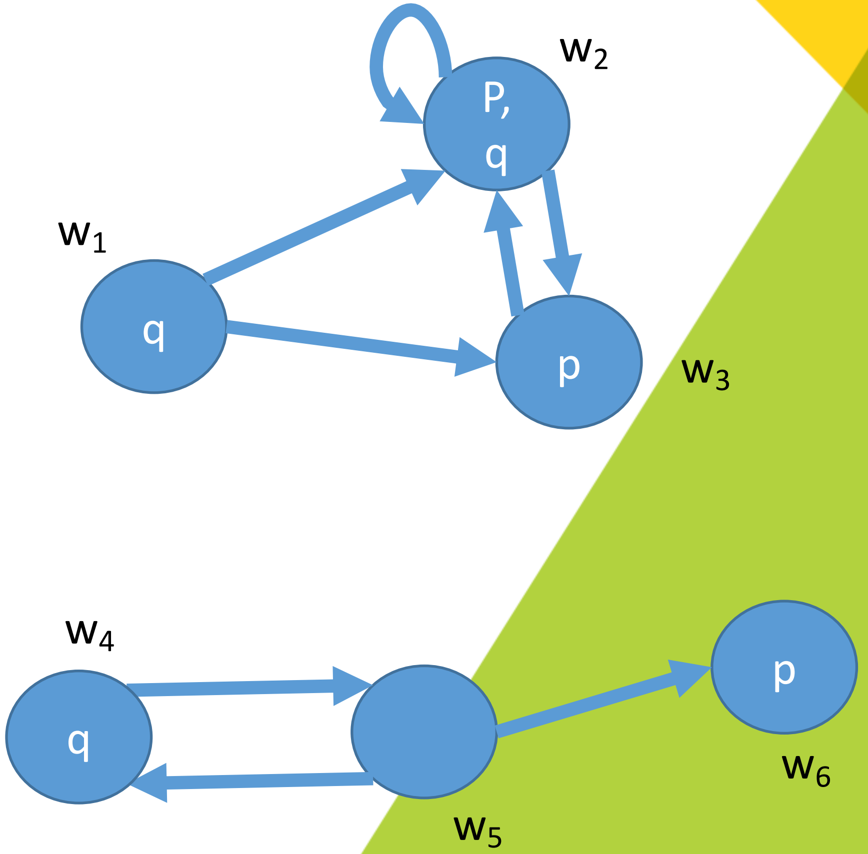
Formula schema: high-level pattern (we've seen this before)

e.g., $\varphi \vee \neg\varphi$ is a schema;

$p \vee \neg p$ and $(p \rightarrow q) \vee \neg(p \rightarrow q)$ are instances

We *want* $\Box p \rightarrow p$ to be true when talking about knowledge, but not belief (even though it isn't valid generally).

When we assert a formula schema, we introduce it as an **axiom.**



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$\diamond \varphi \rightarrow \diamond \square\varphi$	If agent Q doesn't know something, it doesn't know what it doesn't know	If agent Q doesn't believe something, it doesn't believe what it doesn't believe		
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$\Box \varphi \rightarrow \diamond \varphi$	Agent Q can chain knowledge (true things)	Agent Q can chain belief (true things)		

Axiom	Knowledge	Belief	Property (axiom name)	R(w, w')
$\Box p \rightarrow p$	Agent Q only knows true things	NOT A SUITABLE AXIOM	Reflexive (T)	
$\Box \varphi \rightarrow \Box\Box\varphi$	Agent Q knows what it knows (introspection)	Agent Q believes what it believes		
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$\diamond\varphi \rightarrow \Box\diamond\varphi$	If agent Q doesn’t know something, it doesn’t know what it doesn’t know	If agent Q doesn’t believe something, it doesn’t believe what it doesn’t believe		
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Axiom	Knowledge	Belief	Property (axiom name)	R(w, w’)
$\square p \to p$	Agent Q only knows true things	NOT A SUITABLE AXIOM	Reflexive (T)	
$\square \varphi \to \square\square\varphi$	Agent Q knows what it knows (introspection)	Agent Q believes what it believes	Transitive (4)	$\forall(w, w', w'') \in (W \times W \times W), (R(w, w') \land R(w', w'') \to R(w, w''))$
$\diamond \varphi \to \square \diamond \varphi$	If agent Q doesn’t know something, it doesn’t know what it doesn’t know	If agent Q doesn’t believe something, it doesn’t believe what it doesn’t believe	Euclidean (5)	$\forall(w, w', w'') \in (W \times W \times W), (R(w, w') \land R(w, w'') \to R(w', w''))$
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